Part 1: The Buck Converter

In a buck converter, the inductor is in series with the output.

1.1 Output Capacitor (C_{out})

The output capacitor sits directly next to the inductor. It filters the continuous AC ripple, providing a steady DC voltage to the load.

A. RMS Current Derivation

The capacitor blocks the DC load current, leaving a pure, zero-centered triangular AC wave with a peak-to-peak amplitude of ΔI_L.

We model the rising slope over time period T as $i(t) = \left(\frac{\Delta I_L}{T}\right) t$. We integrate this squared current over one segment (from -T/2 to +T/2) to find the mean square.

$$I_{RMS}^2 = \frac{1}{T}\int_{-T/2}^{T/2} \left[\left(\frac{\Delta I_L}{T}\right) t\right]^2 \, dt$$

$$I_{RMS}^2 = \frac{\Delta I_L^2}{T^3}\int_{-T/2}^{T/2} t^2 \, dt$$

$$I_{RMS}^2 = \frac{\Delta I_L^2}{T^3}\left[\frac{t^3}{3}\right]_{-T/2}^{T/2}$$

$$I_{RMS}^2 = \frac{\Delta I_L^2}{T^3}\left[\frac{T^3}{24} + \frac{T^3}{24}\right] = \frac{\Delta I_L^2}{12}$$

Taking the square root yields the final formula:

$$I_{Cout(RMS)} = \frac{\Delta I_L}{\sqrt{12}}$$

B. Minimum Capacitance (C_{min}) Derivation

To find C_{min}, we calculate the charge (ΔQ) entering the capacitor during the positive half of the triangular wave. The area of a triangle is $\frac{1}{2} \cdot \text{base} \cdot \text{height}$.

- Base (Time): $\frac{T}{2}$
- Height (Current): $\frac{\Delta I_L}{2}$

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta I_L}{2}\right) = \frac{\Delta I_L \cdot T}{8}$$

Substitute ΔQ into the fundamental capacitor equation ($C = \frac{\Delta Q}{\Delta V}$) and replace T with $\frac{1}{f_{sw}}$:

$$C_{min} = \frac{\Delta I_L}{8 \cdot f_{sw} \cdot \Delta V_{out}}$$

1.2 Input Capacitor (C_{in})

The input capacitor sits next to the main switch. It provides instantaneous rectangular pulses of current to the inductor when the switch closes.

A. Exact RMS Current Derivation

The current is a rectangular pulse containing the inductor's triangular ripple during the ON time ($D \cdot T$), and zero during the OFF time.

- ON State Current: $I_{out}(1 - D) + i_{ripple}(t)$
- OFF State Current: $-I_{out} \cdot D$

We integrate the squared currents over their respective times. The cross-term (2AB) of the ripple expansion integrates to zero because it is zero-mean.

$$I_{RMS}^2 = \frac{1}{T}\left[\int_0^{DT} \left(I_{out}(1 - D) + i_{ripple}(t)\right)^2 \, dt + \int_{DT}^{T} (-I_{out} \cdot D)^2 \, dt\right]$$

$$I_{RMS}^2 = I_{out}^2 (1 - D)^2 \cdot D + \frac{\Delta I_L^2}{12} \cdot D + I_{out}^2 \cdot D^2 (1 - D)$$

Factoring out $I_{out}^2 \cdot D(1 - D)$ from the ideal terms leaves $(1 - D) + D$, which simplifies to 1. Taking the square root yields:

$$I_{Cin(RMS)} = \sqrt{I_{out}^2 \cdot D(1 - D) + \frac{\Delta I_L^2}{12} \cdot D}$$

B. Minimum Capacitance (C_{min}) Derivation

We calculate the charge (ΔQ) by finding the area of the rectangular charging current during the OFF state.

- Current: Average input current, $I_{in} = I_{out} \cdot D$
- Time: $(1 - D)T$

$$\Delta Q = I_{out} \cdot D \cdot (1 - D) \cdot T$$

Substitute into $C = \frac{\Delta Q}{\Delta V}$ and replace T with $\frac{1}{f_{sw}}$:

$$C_{min} = \frac{I_{out} \cdot D \cdot (1 - D)}{f_{sw} \cdot \Delta V_{in}}$$

Part 2: The Boost Converter

In a boost converter, the inductor is in series with the input.

2.1 Input Capacitor (C_{in})

The input capacitor sits directly next to the inductor. Its role is physically identical to the buck converter's output capacitor: it filters the continuous, zero-centered triangular AC ripple.

Because the waveform is identical, the mathematical derivations are identical to Buck converter.

- **RMS Current:**

$$I_{Cin(RMS)} = \frac{\Delta I_L}{\sqrt{12}}$$

- **Minimum Capacitance:**

2.2 Output Capacitor (C_{out})

$$C_{min} = \frac{\Delta I_L}{8 \cdot f_{sw} \cdot \Delta V_{in}}$$

The output capacitor sits behind the blocking diode. It acts as the sole power source for the load when the main switch is closed.

A. Exact RMS Current Derivation

The current is $-I_{out}$ during the ON time ($D \cdot T$), and a massive recharge pulse containing the inductor ripple during the OFF time $(1 - D)T$).

- ON State Current: $-I_{out}$
- OFF State Current: $I_{out}\left(\frac{D}{1-D}\right) + i_{ripple}(t)$

We integrate the squared currents. Again, the cross-term of the ripple expansion integrates to zero.

$$I_{RMS}^2 = \frac{1}{T}\left[\int_0^{DT} (-I_{out})^2 \, dt + \int_{DT}^{T} \left(I_{out}\left(\frac{D}{1-D}\right) + i_{ripple}(t)\right)^2 \, dt\right]$$

$$I_{RMS}^2 = I_{out}^2 \cdot D + I_{out}^2\left(\frac{D^2}{(1-D)^2}\right)(1-D) + \frac{\Delta I_L^2}{12}(1-D)$$

Factoring the ideal terms ($I_{out}^2 \cdot D + I_{out}^2 \frac{D^2}{1-D}$) simplifies cleanly to $I_{out}^2 \frac{D}{1-D}$. Taking the square root yields:

$$I_{Cout(RMS)} = \sqrt{I_{out}^2 \frac{D}{1-D} + \frac{\Delta I_L^2}{12}(1-D)}$$

B. Minimum Capacitance (C_{min}) Derivation

We calculate the charge (ΔQ) drained from the capacitor during the ON state, when it is entirely isolated from the input.

- Current: The steady load current, I_{out}
- Time: $D \cdot T$

$$\Delta Q = I_{out} \cdot D \cdot T$$

Substitute into $C = \frac{\Delta Q}{\Delta V}$ and replace T with $\frac{1}{f_{sw}}$:

$$C_{min} = \frac{I_{out} \cdot D}{f_{sw} \cdot \Delta V_{out}}$$